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# Probability weighting and loss aversion in futures hedging

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#### Abstract

We analyze how the introduction of probability weighting and loss aversion in a futures hedging model affects decision making. Analytical findings indicate that probability weighting alone always affects optimal hedge ratios, while loss and risk aversion only have an impact when probability weighting exists. In the presence of probability weighting, simulation results for a relevant range of parameter values suggest that probability weighting is dominant; changes in probability weighting affect hedge ratios relatively more than changes in loss and risk aversion. When decisions are made independently, loss aversion has a relatively small impact on hedge ratios for all parameter values, and risk aversion becomes important for only a narrow range of risk coefficients which produce implausible speculative behavior. When prior losses and gains affect behavior, hedging is influenced most by prior outcomes that influence risk attitudes, but this effect is still somewhat less than the consequences of changes in probability weighting. © 2008 Elsevier B.V. All rights reserved.

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Experimental evidence has demonstrated that the assumptions of the standard expected utility theory are often violated when people make decisions under risk. Schoemaker (1982) and Starmer (2000) discuss problems encountered by expected utility theory and new developments in the field of decision making. De Bondt and Thaler (1995), Hirshleifer (2001) and Barberis and Thaler (2003) provide empirical evidence critical of expected utility theory in financial decision making. As a consequence, researchers have developed alternative theories to explain choice, often based on observed laboratory or experimental evidence. In financial applications, prospect theory developed by Kahneman and Tversky (1979) and Tversky and Kahneman (1992) appears to offer the most promising nonexpected utility theory for explaining decision making under risk (Barberis and Thaler, 2003). Prospect theory differs from the expected utility paradigm in that choice is influenced by loss aversion and probability weighting. Loss aversion posits that decisions are made in terms of gains and losses rather than final wealth, and individuals react differently to gains and losses. Probability weighting reflects the notion that decision makers use transformed probabilities rather than objective probabilities in making their choices. The choice model under prospect theory has two fundamental components: a value function that incorporates loss aversion, and a weighting function that reflects a nonlinear transformation of probability.

Although extensive research on these dimensions exists, Barberis and Thaler (2003) assert that most work in behavioral finance is narrow as "models typically capture something about investors' beliefs, or their preferences, or the limits to arbitrage, but not all three" (p. 1112). Empirical research in behavioral finance has been dominated by the investigation of loss aversion. Applied work has focused primarily on either loss aversion and reference points to re-examine unexplained phenomena, or analytically incorporated loss aversion and reference points in investment allocation problems (e.g. Benartzi and Thaler, 1995; Barberis Huang, and Santos, 2001). The work investigating the combined effects of the value and probability functions in financial decision making is only now emerging. Strikingly the research findings show that substantive changes can arise when multiple dimensions of the behavioral literature are considered (Blavatskyy and Pogrebna, 2005; Davies and Satchell, 2005; Langer and Weber, 2005), but the complexity of the phenomena suggests that additional work is needed. Further, most recent research has focused primarily on asset price models for equity and bond markets, with little analysis on the role of behavioral theory in futures markets and the hedging decision. To date, only two papers have investigated hedging decisions in the presence of loss aversion (Albuquerque, 1999; Lien, 2001); neither considered probability weighting. The extension to hedging models may be particularly informative because previous research has suggested that loss aversion has no effect on hedging decisions in the presence of unbiased markets (Lien, 2001). Introduction of probability weights, which may deviate from market or objective probabilities, with loss aversion in an integrated framework may provide a more comprehensive understanding of behavioral dimensions of hedging decisions.

The purpose of this paper is to incorporate loss aversion and probability weighting into a futures hedging context. To identify the effects of these factors, a theoretical model is developed and the importance of loss aversion and probability weighting is discussed. In our empirical study we examine the effect of loss aversion and probability weighting for a soybean producer who hedges his crop using a futures market. Distributions for soybean cash and futures price changes are developed to simulate the producer's operational hedging decision for a relevant range of parameter values. Calculated hedge ratios are used to examine the absolute and relative impacts of loss aversion and probability weighting on hedging decisions.

#### 1. Literature review

Little financial research exists that combines value and probability functions to study behavior. Most applied research has focused on aspects of the value function and demonstrated how models that incorporate loss aversion can produce behavior consistent with observed behavior. For example, Benartzi and Thaler (1995) explore an explanation to the equity premium puzzle based on myopic loss aversion. Using simulations and previously estimated value function parameters, they conclude that the size of the equity premium is consistent with investors exhibiting myopic loss aversion when portfolios are evaluated annually. Barberis, Huang, and Santos (2001) also depart from a strict consumption-based framework to explain the behavior of stock returns. By assuming that investors are loss averse and derive utility from changes in financial wealth as well as consumption, they are able to generate high mean, excess volatility, predictability of stock returns, and low correlation between stock returns and consumption growth. Similarly, Berkelaar and Kouwenberg (2003), who studied an equilibrium asset pricing in which some agents are myopic loss averse, find higher volatility and larger price changes relative to similar models under an expected utility framework. They suggest that their model offers a potential explanation for the erratic stock market in the late 1990s.

Recently several studies have investigated financial behavior using multi-dimensional models that incorporate both value and weighting functions. Levy and Levy (2002) examine whether risk aversion characterizes investors and the effect of probability weights on risk premium. They conclude that risk aversion is not present over the entire wealth domain, and behavior may be explained either by non-concavity of the utility function or the presence of a probability function. They argue that even if the utility function were concave in the entire wealth, individuals could still act as risk-seeking investors due to probability weighting. In some situations, their results indicate that probability weights can enhance risk aversion. Blavatskyy and Pogrebna (2005) and Langer and Weber (2005) extend the analysis of the effect of myopic loss aversion on investment decisions by introducing probability weighting. In the presence of only myopic loss aversion, Berkelaar, Kouwenberg, and Post (2004) and Hwang and Satchell (2005) argue that investors reduce the proportion of risky assets in their portfolios. However, introduction of probability weighting leads to an opposite effect on investment decisions, in some cases completely offsetting the effect of loss aversion (Blavatskyy and Pogrebna, 2005). Similarly, myopic loss-averse investors who also transform probabilities may decide to increase rather than decrease the proportion of risky assets in their portfolios (Langer and Weber, 2005).

In a different approach, Davies and Satchell (2005) integrate the value and weighting functions to study the allocation of investments. Using stock market data on excess returns from the US and UK, they investigate what parameter values of their functions are needed to make their analytical portfolio and observed allocations correspond. Assuming a power function for preferences and Prelec's one-parameter weighting function, the curvature of the value function needs to be greater in the loss domain than in the gain domain. The optimal portfolio is highly sensitive to loss aversion as allocations consistent with observed behavior are only found for values of loss aversion ranging from 1.8 to 2.6. They also

identify that introducing probability weighting tends to decrease the estimated coefficient of loss aversion and narrow the interval of feasible values.

Research that investigates the effects of considering both the value and weighting functions in financial decision making is emerging. Substantive changes appear to arise when multiple dimensions of the behavioral literature are considered, but the complexity of the phenomena point to the need for additional work to increase our understanding of their importance. Further, most recent research has focused primarily on asset price models for equity and bond markets, with little analysis of the role of behavioral theory in futures markets and the hedging decision. To date, only two papers have examined hedging decisions in the presence of loss aversion. Albuquerque (1999) adopts a hedging model for a loss-averse firm to investigate the role of options and forwards in currency hedging strategies. He argues that a loss-averse firm is primarily concerned with downside risk, and thus a hedge is placed only when there is a chance of losses. His results indicate that forward contracts are a better instrument than options to hedge currency risk, which contrasts with standard advice favoring out-of-the-money puts to hedge against downside risk. Lien (2001) examines how the introduction of loss aversion affects the optimal strategy of a short hedger, and finds that loss aversion has no impact on optimal hedge ratios when markets are unbiased. Loss aversion only affects hedging in the presence of contango or backwardation, and the direction and magnitude of the impact depend on the degree of loss and risk aversion. However, simulation results for several levels of loss and risk aversion under different contango and backwardation scenarios show small variability, with numbers ranging from 0.87 to 1.0 for the optimal hedge. No research exists that considers the effect of probability weighting and loss aversion on the hedging decision in futures markets, which is the focus of our work.

#### 2. Non-expected utility models

Schoemaker (1982) argues that expected utility theory fails as a descriptive and predictive framework because it does not recognize psychological principles of judgment and choice. Research has shown that individuals don't structure problems and process information consistent with expected utility theory. Among the alternatives to expected utility, prospect theory is the most successful at capturing the empirical evidence from field and lab experiments (Barberis and Thaler, 2003). The choice model is based on a function V(x) with two components (Eq. (1)): a value function v(x) and a weighting function w(F(x)), where x is the argument of the value function, and F(x) is the objective cumulative probability distribution of x (Rieger and Wang, 2006).

$$V(x) = \int v(x) \frac{\mathrm{d}}{\mathrm{d}x} w(F(x)) \,\mathrm{d}x. \tag{1}$$

The value function takes into account that variations in the framing of alternatives systematically yield different preferences (framing effects), i.e., agents react differently to gains and losses. A loss-aversion coefficient  $\lambda$  is incorporated to account for the fact that losses loom larger than gains, i.e., individuals are more sensitive to losses than to gains. The function also allows for risk-averse behavior (concavity) in the domain of gains (x>0), and risk-seeking behavior (convexity) in the domain of losses (x<0). Risk seeking in the loss domain has received empirical support and arises from the notion that individuals dislike losses to such an extent that they are willing to take greater risks in

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order to make up for their losses (Heisler, 1994; Odean, 1998; Coval and Shumway, 2005). Prior losses or gains can also influence behavior as they affect the utility of subsequent outcomes. The response to prior outcomes is influenced by how decision makers incorporate previous outcomes and whether risk attitudes change (Thaler and Johnson, 1990; Barberis, Huang, and Santos, 2001).

In our study the value function is represented by a constant absolute risk aversion functional form in Eq. (2):

$$v(x) = \begin{cases} 1 - e^{-\Theta_{G}(x)}, & x \ge 0\\ -\lambda \left[1 - e^{\Theta_{L}(-x)}\right], & x \le 0 \end{cases}$$
(2)

where  $\lambda$  is the loss-aversion parameter, and  $\Theta_{\rm G}$  and  $\Theta_{\rm L}$  are the risk-aversion parameters for gains and losses. Empirical research in eliciting value functions usually finds evidence supporting an S-shaped curve with values for the loss-aversion parameter  $\lambda$  generally greater than 2—losses are weighted more than two times as heavily as gains. For instance, based on lab experiments with graduate students, Tversky and Kahneman (1992) estimate loss aversion to be  $\lambda = 2.25$ . Pennings and Smidts (2003) elicit producers' utility functions and found  $\lambda = 2.5$ . Other studies use aggregate market data on returns and a choice model based on the S-shaped value function to estimate loss aversion. Berkelaar, Kouwenberg, and Post (2004) use US stock market data to calculate excess returns, and estimate loss aversion to be 2.71. Davies and Satchell (2005) use US and UK aggregate data on equity excess returns and find that strategies consistent with observed asset allocation can only be obtained when loss aversion is within the interval [1.8, 2.6].

A second component of prospect theory is the weighting function. It was developed from the empirical observation that individuals do not treat probabilities linearly, but rather weight them. The weighting function w(p) adopted here was proposed by Prelec (1998) and is characterized by a unique parameter  $\gamma$  in Eq. (3):

$$w(p) = \exp\left[-(-\ln p)^{\gamma}\right]$$
(3)

where p is the objective probability, and  $\gamma$  defines the curvature of the curve. This function assumes equal probability weighting in the domains of gains and losses. Although other functional forms exist, Prelec's one-parameter function was selected based on work by Gonzalez and Wu (1999) and Stott (2006) who after testing several functional forms conclude that Prelec's one-parameter probability function provides a good and parsimonious fit to experimental data.

Prelec's function is an increasing function of probability p. When  $0 < \gamma < 1$  the probability function is regressive and takes an inverse s-shape. In this situation, initially when the function is concave small probabilities are overweighted (w(p) > p), but when the function becomes convex high probabilities are underweighted (w(p) < p). For  $\gamma > 1$  the probability function is S-shaped which implies that small probabilities are underweighted and high probabilities are overweighted. The function is also asymmetric in the sense that the inflection point, the point at which the function intersects the diagonal given by  $\gamma = 1$ , is at p = w(p) = 0.37.

Empirical evidence tends to support the inverse S-shaped probability function, which implies  $0 < \gamma < 1$ . Using experiments with undergraduate and graduate students, Abdellaoui (2000) and Abdellaoui, Vossmann, and Weber (2005) find values between 0.6 and 0.9 for

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the curvature parameter.<sup>1</sup> However, Alarie and Dionne (2001) argue that the inverse S-shaped curve may not be a proper representation in all situations. Humphrey and Verschoor (2004) conduct experiments with households in rural, farming communities in Uganda, India and Ethiopia, and find evidence consistent with S-shaped weighting function which implies  $\gamma > 1$ .

#### 3. Analytical model

The analysis is based on a soybean producer who harvests and stores soybeans at t = 0when his initial wealth is  $W_0$ . The stored soybeans are sold in the subsequent period (t = 1) at the random cash price  $C_1$ . The producer can trade soybean futures contracts to hedge his cash position, taking a short futures position at futures price  $F_0$  in the initial period (t = 0) and offsetting this position at the random futures price  $F_1$  in the subsequent period (t = 1). In this one-period model final wealth  $W_1$  is given by  $W_1 = W_0 R$ , where  $R = r_c + (1 - r_f)h$ ,  $r_c$  is the return on the cash position,<sup>2</sup>  $r_f$  is the return on the futures position, and h is the hedge ratio. Here the hedge ratio is positive for short positions and negative for long positions. First, consider loss aversion where wealth change,  $W_1 - W_0 = W_0(R - 1)$ , is adopted as the argument of a constant absolute risk aversion function.<sup>3</sup> Taking the expected value of the functional form in (4), a two-moment representation of preferences is given in (5):

$$E[v(W_1 - W_0)] = \int_0^{+\infty} [1 - e^{-\Theta_G(W_1 - W_0)} f(W_1 - W_0) d(W_1 - W_0)] - \lambda \int_{-\infty}^0 [1 - e^{-\Theta_G(W_1 - W_0)} f(W_1 - W_0) d(W_1 - W_0)]$$
(4)

$$E[v(W_1 - W_0)] \equiv V(\mu_h, \sigma_h) = (\mu_h - \frac{W_0 \Theta_G}{2} \cdot \sigma_h^2) + \lambda \left(\mu_h + \frac{W_0 \Theta_L}{2} \cdot \sigma_h^2\right)$$
$$= (\mu_h - \frac{\theta_G}{2} \cdot \sigma_h^2) + \lambda (\mu_h + \frac{\theta_L}{2} \cdot \sigma_h^2)$$
(5)

where  $f(W_1 - W_0)$  is the objective cumulative distribution function of wealth change,  $\mu_h = E(W_1 - W_0) = W_0[\mu_c + (1 - \mu_f)h - 1], \mu_c(\mu_f)$  is the mean of the cash (futures) return distribution,  $\sigma_h^2 = Var(W_1 - W_0) = W_0^2[\sigma_c^2 + h^2\sigma_f^2 - 2h\sigma_{cf}], \sigma_c^2(\sigma_f^2)$  is the variance of the cash (futures) return distribution,  $\sigma_{cf}$  is the covariance between cash and futures returns,  $\theta_G = W_0 \Theta_G$  is the relative risk-aversion coefficient in the domain of gains,  $\theta_L = W_0 \Theta_L$  is the relative risk-aversion coefficient in the domain of losses, and  $\lambda$  is the loss-aversion coefficient.

<sup>&</sup>lt;sup>1</sup>While neither study specifically adopted Prelec's function the results for the curvature parameter are consistent with an inverse *s*-shaped function.

 $<sup>{}^{2}</sup>r_{c} = C_{1}/C_{0}$ , where  $C_{0}$  and  $C_{1}$  are the respective cash prices at the beginning and at the end of the hedging period. The returns on the futures position are calculated similarly.

<sup>&</sup>lt;sup>3</sup>The conceptual model is developed in a mean-variance framework to facilitate the analytical findings. The use of mean-variance is consistent with the normal returns distribution encountered in the data. In the absence of normality, the optimal hedge ratios presented later can be obtained by optimizing (4) with respect to h.

The optimal hedge ratio is obtained by maximizing (5) with respect to the hedge ratio:

$$\frac{\partial}{\partial h} E[v(W_1 - W_0)] = \frac{\partial V(\mu_h, \sigma_h)}{\partial \mu_h} \frac{\partial \mu_h}{\partial h} + \frac{\partial V(\mu_h, \sigma_h)}{\partial \sigma_h} \frac{\partial \sigma_h}{\partial h} = 0$$
(6)

$$h = \frac{(1+\lambda)(1-\mu_{\rm f})}{(\theta_{\rm G}-\lambda\theta_{\rm L})\sigma_{\rm f}^2} + \frac{\sigma_{\rm cf}}{\sigma_{\rm f}^2}.$$
(7)

The hedge ratio in (7) incorporates loss aversion in the speculative component, but to this point there is no probability weighting. In the absence of loss aversion, ( $\lambda = 0$ ), the speculative component in (7) is similar to that found in the standard hedging formulation where returns are specified on a percentage basis. With loss aversion,  $\lambda$  affects both the numerator and denominator of the speculative component. In the numerator,  $\lambda$  magnifies the effect of inefficiencies in the futures market on the optimal hedge. In the denominator, the effect of loss aversion is influenced by the difference in degree of concavity in the loss and gain domains, ( $\theta_{\rm G} - \lambda \theta_{\rm L}$ ), reflecting the effect of the overall shape of the value function on hedging activity. In the absence of probability weighting, when the futures market is unbiased ( $\mu_{\rm f} = 1$ ) the speculative component of the hedge ratio is zero, and the optimal hedge ratio is the minimum-variance hedge ratio given by $\sigma_{\rm cf}/\sigma_{\rm f}^2$ . This finding is consistent with Lien (2001), who finds that loss aversion has no effect on the optimal hedge ratio when the futures market is unbiased.<sup>4</sup>

Now introduce probability weighting in the hedging problem. The optimal hedge ratio can be derived in a similar fashion, except that transformed probabilities—and not objective probabilities—are used in the calculation of means and variances. Thus the optimal hedge ratio with probability weighting is given by (8):

$$h_{\rm p} = \frac{(1+\lambda)(1-\mu_{\rm f,p})}{(\theta_{\rm G}-\lambda\theta_{\rm L})\sigma_{\rm f,p}^2} + \frac{\sigma_{\rm cf,p}}{\sigma_{\rm f,p}^2}$$
(8)

where  $h_p$  is the hedge ratio,  $\mu_{f,p}$  and  $\sigma_{f,p}^2$  are the mean and variance of the transformed distribution of futures returns, and  $\sigma_{cf,p}$  is the covariance between cash and futures returns.

The moments of the distributions with probability weighting are given by:

$$\mu_{\rm f,p} = \mu_{\rm f} + \Delta \mu_{\rm f} \tag{9}$$

$$\sigma_{\rm f,p}^2 = \sigma_{\rm f}^2 - \mu_{\rm f}^2 + \{\mu_{\rm f} + \Delta\mu_{\rm f}\}^2 - 2\Delta\sigma_{\rm f}^2$$
(10)

$$\sigma_{\rm cf,p} = \sigma_{\rm cf} + \Delta \rho - \mu_{\rm f} \Delta \mu_{\rm c} - \mu_{\rm c} \Delta \mu_{\rm f} - \Delta \mu_{\rm f} \Delta \mu_{\rm c} \tag{11}$$

where

$$\Delta \mu_{\rm f} = \int_{-\infty}^{+\infty} F(r_{\rm f}) \,\mathrm{d}r_{\rm f} - \int_{-\infty}^{+\infty} w[F(r_{\rm f})] \,\mathrm{d}r_{\rm f} \tag{12}$$

$$\Delta\mu_{\rm c} = \int_{-\infty}^{+\infty} F(r_{\rm c}) \,\mathrm{d}r_{\rm c} - \int_{-\infty}^{+\infty} w[F(r_{\rm c})] \mathrm{d}r_{\rm c} \tag{13}$$

<sup>&</sup>lt;sup>4</sup>He also finds that when markets are in contango or backwardation the optimal hedge ratio in the presence of loss aversion will differ from the minimum-variance hedge ratio.

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$$\Delta \sigma_{\rm f}^2 = \int_{-\infty}^{+\infty} r_{\rm f} F(r_{\rm f}) \,\mathrm{d}r_{\rm f} - \int_{-\infty}^{+\infty} r_{\rm f} w[F(r_{\rm f})] \,\mathrm{d}r_{\rm f} \tag{14}$$

$$\Delta \rho = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} F(r_{\rm f}, r_{\rm c}) \,\mathrm{d}r_{\rm f} \,\mathrm{d}r_{\rm c} - \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} w[F(r_{\rm f}, r_{\rm c})] \,\mathrm{d}r_{\rm f} \,\mathrm{d}r_{\rm c}. \tag{15}$$

In Eqs. (9) through (15),  $r_c(r_f)$  is the cash (futures) return,  $F(r_c)$  and  $F(r_f)$  are respectively the objective cumulative distribution functions of cash and futures returns,  $F(r_c, r_f)$  is the objective joint cumulative distribution function of cash and futures returns,  $w[F(r_c)]$  and  $w[F(r_f)]$  are respectively the transformed cumulative distribution functions of cash and futures returns, and  $w[F(r_c, r_f)]$  is the transformed joint cumulative distribution function of cash and futures returns.<sup>5</sup> In the absence of probability weighting,  $\Delta \mu_f = \Delta \mu_c = \Delta \sigma_f^2 =$  $\Delta \rho = 0$  and consequently  $\mu_{f,p} = \mu_f$ ,  $\sigma_{f,p}^2 = \sigma_f^2$ , and  $\sigma_{cf,p} = \sigma_{cf}$ .

The hedge ratio in (8) incorporates both probability weighting and loss aversion, and probability weighting is present in the speculative and hedging components. As a result the speculative component of the hedge ratio does not vanish when the futures market is unbiased, and loss aversion will have an effect on the optimal hedge.

#### 3.1. Loss aversion and risk aversion effects

Since (8) indicates that loss aversion does not disappear in the presence of unbiased futures, consider in (16) and (17) the derivatives of (8) with respect to the loss-aversion coefficient  $\lambda$ . The sign of (16) is given by the sign of the expected futures return  $(1 - \mu_{\rm fp})$ , because all other parameters are positive, and the sign of (17) depends on the sign of  $(1 - \mu_{f,p})$  and  $(\theta_G - \lambda \theta_L)$ . By definition, the loss-aversion parameter is greater than one  $(\lambda > 1)$  and the curvature parameters (risk-aversion coefficients) are positive ( $\theta_G, \theta_L > 0$ ). Tversky and Kahneman (1992) estimated the curvature parameters in the domains of gains and losses and found that they are equal in value. Subsequent studies that have investigated loss aversion in this framework have assumed the curvature parameters are the same in the gain and loss domains (Lien, 2001; Berkelaar, Kouwenberg, and Post, 2004; Langer and Weber, 2005). Recently, Davies and Satchell (2005) modeled investment allocation in a prospect theory framework and searched for parameter values needed to make their model consistent with observed behavior. They found that the curvature parameters need to be close in value, with the curvature in the loss domain greater than in the gain domain. Here, based on the most recent findings, we assume that  $\theta_L > \theta_G$  and only examine situations for which their difference is small, implying that  $(\theta_{\rm G} - \lambda \theta_{\rm L}) < 0.6$ 

$$\frac{\partial h_{\rm p}}{\partial \lambda} = \frac{(1 - \mu_{\rm f,p})(\theta_{\rm G} + \theta_{\rm L})}{(\theta_{\rm G} - \lambda \theta_{\rm L})^2 \sigma_{\rm f,p}^2} \tag{16}$$

$$\frac{\partial^2 h_{\rm p}}{\partial \lambda^2} = \frac{2\theta_{\rm L}(1-\mu_{\rm f,p})(\theta_{\rm G}+\theta_{\rm L})}{(\theta_{\rm G}-\lambda\theta_{\rm L})^3\sigma_{\rm f,p}^2}.$$
(17)

<sup>&</sup>lt;sup>5</sup>In the analysis to follow recall that the transformed CDFs are given by (3) where  $\gamma$  characterizes the degree of probability weighting.

<sup>&</sup>lt;sup>6</sup>When the curvature parameters are the same in the gain and loss domains ( $\theta_G = \theta_L = \theta$ ), ( $\theta_G - \lambda \theta_L$ ) =  $\theta(1 - \lambda) < 0$ .

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When  $(1 - \mu_{f,p}) > 0$ ,  $\partial h_p / \partial \lambda > 0$  and  $\partial^2 h_p / \partial \lambda^2 < 0$ , which implies a positive relationship between hedge ratio and loss aversion. This relationship becomes less positive as loss aversion increases. When  $(1 - \mu_{f,p}) < 0$ ,  $\partial h_p / \partial \lambda < 0$  and  $\partial^2 h_p / \partial \lambda^2 > 0$ , which implies a negative relationship between the hedge ratio and loss aversion that becomes less negative as loss aversion increases. Thus,  $\partial h_p / \partial \lambda$  tends to zero as the loss-aversion coefficient  $\lambda$ becomes large. Intuitively, since  $\lambda$  only appears in the speculative component of the optimal hedge Eq. (8), producers with a high degree of loss aversion are less willing to speculate on the returns from the futures position.

Similarly, risk aversion impacts only the speculative dimension of hedging through the term ( $\theta_{\rm G} - \lambda \theta_{\rm L}$ ). Consider in (18) and (19) the derivatives of (8) with respect to ( $\theta_{\rm G} - \lambda \theta_{\rm L}$ ). The sign of (18) is determined by the expected futures return  $(1 - \mu_{\rm f,p})$ , because all other parameters are positive, and the sign of (19) depends on the sign of  $(1 - \mu_{\rm f,p})$  and  $(\theta_{\rm G} - \lambda \theta_{\rm L})$ . Recall that the term  $(\theta_{\rm G} - \lambda \theta_{\rm L})$  is expected to be negative, it increases (decreases) when  $\theta_{\rm G}$  increases (decreases) or  $\theta_{\rm L}$  decreases (increases). When  $(1 - \mu_{f,p}) > 0$ ,  $\partial h_p / \partial (\theta_{\rm G} - \lambda \theta_{\rm L}) < 0$  and  $\partial^2 h_p / \partial (\theta_{\rm G} - \lambda \theta_{\rm L})^2 < 0$ , which implies a negative relationship between the hedge ratio and  $(\theta_{\rm G} - \lambda \theta_{\rm L})$  that becomes more (less) negative as  $(\theta_{\rm G} - \lambda \theta_{\rm L})$  increases (increases) indicating a more (less) risk averse hedger, or if  $\theta_{\rm L}$  decreases (increases) indicating a less (more) risk seeking hedger. When  $(1 - \mu_{f,p}) < 0$ ,  $\partial h_p / \partial (\theta_{\rm G} - \lambda \theta_{\rm L})^2 > 0$ , which implies a positive relationship between the hedge ratio and  $(\theta_{\rm G} - \lambda \theta_{\rm L})^2 > 0$ , which implies a simpler (larger) short position if  $\theta_{\rm G}$  increases (decreases) indicating a more (less) risk averse hedger, or if  $\theta_{\rm L}$  decreases (increases) indicating a less (more) risk seeking hedger. When  $(1 - \mu_{f,p}) < 0$ ,  $\partial h_p / \partial (\theta_{\rm G} - \lambda \theta_{\rm L})^2 > 0$ , which implies a positive relationship between the hedge ratio and  $(\theta_{\rm G} - \lambda \theta_{\rm L})^2 > 0$ , which implies a positive relationship between the hedge ratio and  $(\theta_{\rm G} - \lambda \theta_{\rm L})^2 > 0$ , which implies a positive relationship between the hedge ratio and  $(\theta_{\rm G} - \lambda \theta_{\rm L})^2 > 0$ , which implies a positive as  $(\theta_{\rm G} - \lambda \theta_{\rm L})$  increases (decreases).

$$\frac{\partial h}{\partial(\theta_{\rm G} - \lambda\theta_{\rm L})} = -\frac{(1+\lambda)(1-\mu_{\rm f,p})}{(\theta_{\rm G} - \lambda\theta_{\rm L})^2 \sigma_{\rm f,p}^2}$$
(18)

$$\frac{\partial^2 h}{\partial (\theta_{\rm G} - \lambda \theta_{\rm L})^2} = \frac{2(1+\lambda)(1-\mu_{\rm f,p})}{(\theta_{\rm G} - \lambda \theta_{\rm L})^3 \sigma_{\rm f,p}^2}.$$
(19)

#### 3.2. Probability weighting effect

In contrast to the loss and risk aversion effects, probability weighting which affects the moments of the distribution influences both speculative and hedging components of the hedge ratio. Partial derivatives of (8) with respect to the degree of probability weighting  $\gamma$  yield (20) and (21). Signs of these expressions depend on the signs of the derivatives of the moments with respect to the degree of probability weighting  $\gamma$ , and also on their relative magnitude. An *a priori* effect of probability weighting on the hedge ratio is unclear, which identifies the need for a simulation to investigate the probability weighting effect on hedging decisions.

$$\frac{\partial h_{\rm p}}{\partial \gamma} = \frac{(1+\lambda)}{(\theta_{\rm G}-\lambda\theta_{\rm L})\sigma_{\rm f,p}^2} \left[ \frac{\partial(1-\mu_{\rm f,p})}{\partial \gamma} - \frac{(1-\mu_{\rm f,p})}{\sigma_{\rm f,p}^2} \frac{\partial\sigma_{\rm f,p}^2}{\partial \gamma} \right] + \frac{1}{\sigma_{\rm f,p}^2} \left[ \frac{\partial\sigma_{\rm cf,p}}{\partial \gamma} - \frac{\sigma_{\rm cf,p}}{\sigma_{\rm f,p}^2} \frac{\partial\sigma_{\rm f,p}^2}{\partial \gamma} \right]$$
(20)



Fig. 1. CDF and PDF of cash returns without probability weighting (dark line) and with probability weighting (grey line,  $\gamma = 0.5$ ).

$$\frac{\partial^{2} h_{p}}{\partial \gamma^{2}} = \frac{(1+\lambda)}{(\theta_{\rm G} - \lambda \theta_{\rm L}) \sigma_{\rm f,p}^{2}} \left\{ -\frac{2}{\sigma_{\rm f,p}^{2}} \frac{\partial \sigma_{\rm f,p}^{2}}{\partial \gamma} \cdot \frac{\partial (1-\mu_{\rm f,p})}{\partial \gamma} + \frac{(1-\mu_{\rm f,p})}{(\sigma_{\rm f,p}^{2})^{2}} \left[ 2 \left( \frac{\partial \sigma_{\rm f,p}^{2}}{\partial \gamma} \right)^{2} - \frac{\partial^{2} \sigma_{\rm f,p}^{2}}{\partial \gamma^{2}} \right] \right. \\ \left. + \frac{\partial^{2} (1-\mu_{\rm f,p})}{\partial \gamma^{2}} \right\} + \frac{1}{\sigma_{\rm f,p}^{2}} \left\{ -\frac{2}{\sigma_{\rm f,p}^{2}} \frac{\partial \sigma_{\rm f,p}^{2}}{\partial \gamma} \frac{\partial \sigma_{\rm cf,p}}{\partial \gamma} + \frac{\sigma_{\rm cf,p}}{\sigma_{\rm f,p}^{2}} \left[ \frac{2}{\sigma_{\rm f,p}^{2}} \left( \frac{\partial \sigma_{\rm f,p}^{2}}{\partial \gamma} \right)^{2} - \frac{\partial^{2} \sigma_{\rm f,p}^{2}}{\partial \gamma^{2}} \right] \\ \left. + \frac{\partial^{2} \sigma_{\rm cf,p}}{\partial \gamma^{2}} \right\}.$$

$$(21)$$

#### 4. Simulation design

The empirical simulations are based on futures and cash price changes for soybeans from January 1990 through June 2004 in Illinois.<sup>7</sup> A 4-week hedging horizon is assumed and prices correspond to midweek (Wednesday) closing prices. Overall, the empirical distributions are slightly leptokurtic, but the Jarque-Bera test fails to reject normality for the cash price changes. The *p*-value of the test statistic for futures price change is 0.0571, which is not strong evidence against normality.

In light of the Jarque-Bera test results, means and variances of the sample are used to simulate the distributions for the objective futures and cash returns. Consistent with most research on agricultural futures markets (Garcia and Leuthold, 2004), it is assumed that futures markets are unbiased. The distributions for cash and futures returns have mean 1.00579 and standard deviation 0.06662, and mean 1 and standard deviation 0.06809. The correlation between cash and futures returns is 0.918.

Using these distributions we introduce a weighting function and investigate how different levels of probability weighting affect the values for the expected mean, variance, and covariance. We then investigate the impact of probability weighting and loss aversion on a producer's hedge ratio.

<sup>&</sup>lt;sup>7</sup>Prices were obtained from the Commodity Research Bureau (CRB). Cash prices are for Central Illinois, and futures prices are from the Chicago Board of Trade (CBOT). The nearby futures contract that corresponds to the length of the hedging horizon was used in the analysis.

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Fig. 2. CDF and PDF of cash returns without probability weighting (dark line) and with probability weighting (grey line,  $\gamma = 0.5$ ).



Fig. 3. Expected returns, variance and covariance for different levels of probability weighting (7-gamma).

#### 5. Results

First we examine how probability weighting changes the distribution functions and the moments of the distribution. When  $0 < \gamma < 1$  small probabilities are overweighted and high probabilities are underweighted, yielding a cumulative distribution function (CDF) with more mass in the negative domain and less mass in the positive domain than the original CDF. The derived probability distribution function (PDF) has more mass in the tails and less mass around the mean than the original PDF (Fig. 1). Alternatively, when  $\gamma > 1$  small probabilities are underweighted and high probabilities are overweighted. In this situation probability weighting alters the CDF by decreasing the mass in the negative domain and increasing the mass in the positive domain which implies a PDF which is less dense in the tails and more dense around the mean (Fig. 2).

The effect of probability weighting on the probability distribution impacts the expected returns on cash and futures and their variances. When  $0 < \gamma < 1$  expected returns become higher as  $\gamma$  moves away from 1, but begin to decrease as  $\gamma$  gets close to 0 (Fig. 3). In

#### ARTICLE IN PRESS F. Mattos et al. / Journal of Financial Markets I (IIII) 1.1 0.9 0.7 nedge ratio 0.5 0.3 0.1 0.7 0.5 0.6 0.8 0.9 1 1.1 1.2 1.3 -0.1 -0.3 -0.5 gamma

Fig. 4. Hedge ratio for different levels of probability weighting ( $\gamma$ —gamma). Loss-aversion coefficient  $\lambda = 2.2$ , risk coefficients  $\theta_G = 2$  and  $\theta_L = 2.5$ .

addition, the variance of returns is greater than the true variance for all values of  $\gamma$  in this interval. In contrast, when  $\gamma > 1$  the hedger consistently expects returns to be smaller than the mean of the distribution, and the variance to be smaller than the true variance (Fig. 3). The covariance between cash and futures returns also is affected by probability weighting. In the absence of probability weighting ( $\gamma = 1$ ) the covariance is 0.00416. However, the expected covariance decreases even for small deviations from  $\gamma = 1$ . When  $\gamma$  is 0.7 the expected covariance reaches -0.00026. When  $\gamma$  takes values greater than one, the expected covariance increases (Fig. 3).

Because the moments of the distribution of cash and futures returns are affected by probability weighting, hedge ratios will change. Fig. 4 shows the hedge ratio calculated for different values of  $\gamma$  within a relevant range often reported in empirical studies. In the absence of probability weighting ( $\gamma = 1$ ) the hedge ratio is 0.898, but changes quickly when  $\gamma$  deviates from one. As  $\gamma$  becomes smaller than one, the hedge ratio decreases and reaches zero at  $\gamma = 0.6$ , approximately. As  $\gamma$  becomes larger than one, the hedge ratio increases and appears to level off at the boundary of the relevant range ( $\gamma = 1.3$ ).<sup>8</sup> In Fig. 4 loss and riskaversion parameters are  $\lambda = 2.2$ ,  $\theta_{\rm G} = 2$ , and  $\theta_{\rm L} = 2.5$ . The values of the loss and riskaversion parameters are averages based on previous studies. For example, Pennings and Smidts (2003) estimate  $\lambda = 2.5$  when eliciting agricultural producers' utility functions, and Davies and Satchell (2005) also find coefficients of loss aversion between 1.8 and 2.6 when investigating decisions on asset allocation in equity markets. Lence (1996) uses three levels of relative risk aversion for Iowa grain farmers and classified them as low ( $\theta = 1$ ), moderate ( $\theta = 3$ ) and extremely high ( $\theta = 10$ ), while Nelson and Escalante (2004) find coefficients of relative risk aversion derived from historical financial attributes of Illinois farmers to range from 0.27 to 4.95, with an average value slightly less than 3.9

<sup>&</sup>lt;sup>8</sup>Beyond the boundary, the hedge ratio also declines quickly and approaches zero at  $\gamma = 2$ .

<sup>&</sup>lt;sup>9</sup>These estimates of relative risk aversion were developed using a standard expected utility framework that does not differentiate between gain and loss domains implied by prospect theory.

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Risk aversion		Loss aversion ( $\lambda$ )	Probability weighting $(\gamma)$	Hedge ratio	
$\theta_{\rm G}$	$\theta_{\rm L}$				
2.0	2.5	2.2	0.9	0.7495	
2.0	2.5	2.42 (+10%)	0.9	0.7353	(-1.89%)
2.2 (+10%)	2.5	2.2	0.9	0.7607	(+1.50%)
2.0	2.75 (+10%)	2.2	0.9	0.7243	(-3.35%)
2.0	2.5	2.2	1.0 (+10%)	0.8982	(+19.84%)
2.0	2.5	1.715 (-22%)	0.9	0.8047	(+7.37%)
1.56 (-22%)	2.5	2.2	0.9	0.7288	(-2.76%)
2.0	1.95 (-22%)	2.2	0.9	0.8473	(+13.05%)
2.0	2.5	2.2	0.7 (-22%)	0.2845	(-62.04%)

Table 1									
Effect of probability	weighting,	and	risk	and	loss	aversion	on	hedge	ratios

In each row the bold change in either risk aversion, loss aversion or probability weighting affects the bold hedge ratio, and provides a percentage change.



Fig. 5. Relationship between hedge ratio, probability weighting ( $\gamma$ —gamma) and loss aversion ( $\lambda$ ). Risk coefficients  $\theta_G = 2$  and  $\theta_L = 2.5$ .

The effect of loss and risk aversion on hedge ratios is small relative to the impact of probability weighting. Table 1 shows a comparative analysis of how changes in one parameter at a time affect the hedge ratio. The upper part of Table 1 shows the effect of a 10% increase in each parameter on the hedge ratio relative to the base scenario represented by the first row in italic. A 10% increase in the loss and risk-aversion parameters causes a change of approximately 1-3% in the hedge ratio, with the level of probability weighting held constant. But when a 10% increase is applied to  $\gamma$  the hedge ratio changes by almost

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Fig. 6. Relationship between hedge ratio, probability weighting ( $\gamma$ —gamma), and risk component ( $\theta_G - \lambda \theta_L$ ). Loss aversion coefficient  $\lambda = 2.2$ .

20%. Similarly, the lower part of Table 1 shows that a decrease in loss and risk aversion has a smaller impact on the hedge ratio compared to a decrease of similar magnitude in  $\gamma$ .

However, standard sensitivity analysis should be interpreted with care. Percentage changes in the loss and risk-aversion parameters may not be directly comparable to percentage changes in the probability weighting parameter. We therefore investigate the sensitivity of the findings for the span of values reported in empirical studies.<sup>10</sup> Fig. 5 demonstrates how the hedge ratio changes at different combinations of loss aversion and probability weighting, given risk-aversion parameters  $\theta_G = 2$  and  $\theta_L = 2.5$ . Based on the range of values commonly observed, probability weighting is a dominant factor in the hedging decision while the effect for even large changes in loss aversion is small. Fig. 6 shows how the hedge ratio is influenced by different combinations of the term ( $\theta_G - \lambda \theta_L$ ) and probability weighting when  $\lambda = 2.2$ . Probability weighting continues to dominate while the effect of risk aversion appears to be relevant only when the term ( $\theta_G - \lambda \theta_L$ ) becomes less negative and approaches -1. This happens when both risk-aversion

<sup>&</sup>lt;sup>10</sup>The empirical studies discussed earlier suggest values between 1.8 and 2.6 for loss aversion, between 0.5 and 1.3 for the degree of probability weighting, and between 1 and 5 for risk aversion. The findings presented in Figs. 5 and 6 in the text are qualitatively similar for other relevant values of  $\theta_G$  and  $\theta_L$ , and  $\lambda$  reported in the literature.

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coefficients are extremely small and nearly equal in value or when  $\theta_{\rm G}$  exceeds  $\theta_{\rm L}$ . In this situation, the speculative component of the hedge ratio dominates as the producer assumes a long position in the futures market as well as a long position in the cash market. While analytically feasible, the importance of this finding must be questioned since it is not supported by observed producer speculative behavior in these markets and is inconsistent with the very notion that producers use futures markets to hedge price risk (Lence, 1996; Garcia and Leuthold, 2004).

#### 5.1. Effect of prior gains and losses

To this point we have assumed that the hedger evaluates each transaction separately, implying that outcomes from one period have no effect on decisions in subsequent periods. However, previous losses or gains can influence hedger behavior as they affect the utility of subsequent outcomes.<sup>11</sup> While previous outcomes can affect behavior, the nature of the response can vary depending on how decision makers incorporate previous outcomes and whether risk attitudes change. When decision makers integrate the outcomes of sequential risky choices, the original loss-aversion structure hypothesized by Kahneman and Tversky (1979) that prior losses increase risk-taking, and prior gains reduce it, holds. In effect, the structure of the value function (convex in the loss domain and concave in the gain domain) leads investors to gamble and seek risk when faced with possible losses, and to avoid risk when gains are certain. However, losses or gains may also change decision makers' willingness to take risks. Based on experimental observations, Thaler and Johnson (1990) find evidence that initial gains cause an increase in risk seeking and argue that this can occur because integration of subsequent outcomes is not necessarily sequential or automatic. In an experimental business context, Keasey and Moon (1996) also find prior gains shift behavior towards risk seeking, but no evidence that prior losses shift risk aversion. Barberis, Huang, and Santos (2001) use these findings to develop a model to explain the size of equity premiums and volatility, arguing that previous gains reduce investors' sensitivity to risk while previous losses, by making new losses more painful, increase risk aversion. Recently, Massa and Simonov (2005) find empirical support for the notion that prior gains increase risk taking and prior losses reduce it in an analysis of actual investor behavior.

We examine the effects of prior gains and losses using the original loss-aversion framework proposed by Kahneman and Tversky (1979), and allowing for changes in risk aversion. Following Barberis, Huang, and Santos (2001), we permit previous outcomes to affect the hedger's decisions in a loss aversion framework. The general intuition is that previous gains make losing in the next period somewhat less painful particularly if losses are small, while previous losses make losing in the subsequent period more painful. In our structure, these changes are reflected in a relative sense by conditioning the value of  $\lambda$ , the loss aversion coefficient, on whether previous market activity resulted in a gain or loss. Given  $\lambda$ , a gain (loss) in the previous period makes the hedger's loss aversion decrease (increase) in the subsequent period. The larger the prior loss the more  $\lambda$  assumes its conventional role of heightening the disutility of losses. Changes in risk aversion consistent with observed dynamic behavior are also included by permitting heightened risk aversion (making the value function less concave in the gain domain and more convex in the loss

<sup>&</sup>lt;sup>11</sup>We thank an anonymous reviewer for calling our attention to this point.

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	Risk aversio	n	Loss aversion	Hedge rati	
	$\theta_{ m G}$	$\theta_{\rm L}$	λ		
Prior gain	2.00	2.50	1.80	0.7912	
-	2.00	2.50	2.00	0.7668	
Initial values	2.00	2.50	2.20	0.7495	
Prior loss	2.00	2.50	2.40	0.7365	
	2.00	2.50	2.60	0.7263	
Prior gain	1.50	3.00	1.80	0.7097	
	1.75	2.75	2.00	0.7263	
Initial values	2.00	2.50	2.20	0.7495	
Prior loss	2.25	2.25	2.40	0.7829	
	2.50	2.00	2.60	0.8344	

Table 2				
Hedge ratio for	prior	losses	and	gains

Probability weighting  $\gamma = 0.9$ .

domain) after gains, and more risk aversion (making the value function more concave in the gain domain and less convex in the loss domain) after losses. We investigate the effects of these changes on hedge ratios by allowing  $\lambda$ , and risk coefficients for the loss and gain domains,  $\theta_{\rm L}$  and  $\theta_{\rm G}$ , to fluctuate systematically in response to gains and losses.

The effect of prior outcomes on the hedging decision in the Kahneman and Tversky formulation is implicitly shown in Fig. 5 by simply changing  $\lambda$ , and is presented in the upper part of Table 2 to highlight the relationship. We focus on a probability weighting of  $\gamma = 0.9$  as the results do not differ qualitatively for other levels of  $\gamma$ . At a starting value of  $\lambda = 2.2$  the hedge ratio is 0.75. Allowing the value of  $\lambda$  to change through its relevant range, hedging behavior changes modestly. After a gain the individual becomes less loss averse and the hedge ratio increases to 0.79 ( $\lambda = 1.8$ ). After a loss the individual is more risk seeking and the hedge ratio declines to 0.73 ( $\lambda = 2.6$ ). The pattern of change in the hedging position is consistent with Kahneman and Tversky's original formulation—risk seeking in order to make up for prior losses makes an individual less likely to hedge.

Simulations of changing risk aversion in response to prior losses or gains is observed by allowing the two risk-aversion coefficients,  $\theta_L$  and  $\theta_G$ , and the loss aversion parameter,  $\lambda$ , to fluctuate.  $\lambda$  varies through its relevant range identified in the literature, while  $\theta_L$  and  $\theta_G$  are allowed to move as much as 0.5 in either direction of their starting values. For example, for a large gain which would lead to risk-seeking behavior,  $\theta_L$  changes to 1.5 and  $\theta_L$  to 3.0, reflecting a decrease in concavity in the gain domain and an increase in convexity in the loss domain. The magnitude of the fluctuations of the risk coefficients is coherent with most experimental findings that the coefficients do not differ appreciably and with Davies and Satchell (2005) who find that these parameters need to be close in value to reflect observed behavior. In our simulation, the range of changes in the risk coefficients is most representative for situations in which the losses are not sufficiently catastrophic to result in liquidation of the farm enterprise, which is consistent with the size of most hedge positions relative to farm worth (Collins, 1997). When we allow these parameters to vary we observe behavior consistent with risk seeking after gains and risk aversion after losses. The range of the optimal hedge response (0.1347 from 0.8344 to 0.7097) more than doubles from the

response under the original Kahneman and Tversky formulation (0.0649 from 0.7912 to 0.7263). Beyond the range of risk coefficients, the hedge ratio is fairly stable except when  $\theta_G \ge \theta_L$ . In this situation, large and abrupt changes in hedging behavior occur similar to those depicted in Fig. 6. The price hedger begins to speculate by taking large long positions in the futures market while maintaining long cash positions which as we argued earlier is not consistent with observed producer behavior. Although in a different context, the findings are similar in tone to those encountered by Davies and Satchell (2005) who observe that curvature parameters need to be close in value, with the curvature in the loss domain greater than in the gain domain, for their prospect model to be consistent with observed behavior.

#### 6. Conclusion and discussion

This paper investigates how probability weighting, loss aversion and risk aversion affect decision making in a hedging context. The analytical findings indicate that probability weighting alone always affects optimal hedge ratios, while loss and risk aversion only have an impact when probability weighting exists. In the presence of probability weighting, simulation results based on a relevant range of parameter values suggest that probability weighting is dominant; changes in probability weighting affect hedge ratios relatively more than changes in loss and risk aversion. In a static context, loss aversion has a relatively small impact on hedge ratios for all parameter values, while risk aversion only becomes important for a limited range of risk coefficients, resulting in implausible speculative behavior in the futures market as well as the long position in the cash market. In a situation where prior losses and gains can affect behavior, hedging behavior is influenced most by prior outcomes that influence risk attitudes. Within the relevant range of loss and risk coefficients, the hedge ratio can fluctuate between 0.83 and 0.70 which is larger than in a static context but somewhat less than the effects of changes in probability weighting.

How do our findings relate to actual producer behavior? Empirical research on the observed behavior of producers indicates that they do not speculate or use futures markets to hedge price risk to a large degree. Surveys report that only between 2 and 10 percent of producers use futures markets for hedging (Garcia and Leuthold, 2004). Given that futures markets are efficient and that loss and risk aversion have somewhat limited impact in our simulations, this suggests that many producers have probability weighting functions that deviate from objective market values. This notion is supported by Eales, Engel, Hauser, and Thomson (1990) who compare historical market prices and options-derived distributions to elicited producer subjective expected price distributions. They find that producers tend to slightly overestimate the mean of subsequent prices but severely underestimate their variance. In effect, many grain producers may be overconfident in that they expect prices in excess of those reflected in the futures markets and place little weight in the tails of the distribution. The overconfidence may be reducing their use of futures markets for hedging price risk which they perceive to only a limited degree.<sup>12</sup>

Our results complement the current dialogue in the literature in several ways. First, consistent with previous research (Berkelaar, Kiouwenber, and Post, 2004; Davies and Satchell, 2005; Gomes, 2005; Hwang and Satchell, 2005), our findings support the notion

<sup>&</sup>lt;sup>12</sup>Transaction costs, capital constraints, government programs, and sensitivity to downside risk can also play a role in influencing producer hedging behavior (Garcia and Leuthold, 2004).

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that the introduction of loss aversion in choice models affects the strategies recommended. In contrast to Lien (2001), we find that loss aversion does influence hedging in the presence of efficient markets but only when probability weighting exists. The sensitivity of choice to loss aversion is somewhat limited, but increases with changing risk aversion in response to prior gains and losses. In part the small effect here may be related to the efficiency that characterizes these markets. In the presence of efficiency, the loss aversion effect which is linked to hedge ratios by the speculative component is reduced as the expected return from futures is close to zero. Second, our simulation documents the strength of the impact of probability weighting. Even a small degree of probability weighting in the decision process can drive the optimal hedge ratio significantly from the standard minimum variance estimate. Overall the importance of our probability weighting effect is highly consistent with recent work by Blavatskyy and Pogrebna (2005), Langer and Weber (2005), and Davies and Satchell (2005) that also suggest behavior can change dramatically in its presence.

Our findings which support the importance of probability weighting may provide insight into several recent findings, and suggest directions for further research. Faulkender (2005) investigates whether firms with new debt issuances were actually hedging or just timing the market when selecting their interest rate exposure. He concludes that the firms' riskmanagement practices are driven mainly by speculation or myopia, which is in line with the notion that probability weighting plays a major role in decision-making. Further, behavioral biases such as the endowment effect appear to be less pronounced among experienced traders (List, 2003, 2004). In light of our findings, it seems likely that more experienced traders understand better how to gather and interpret market information, which reduces the degree of probability weighting, and the effects of behavioral biases. Finally the presence of behavioral anomalies like the disposition effect has been empirically detected in micro-level datasets (Heisler, 1994; Odean, 1998; Coval and Shumway, 2005; Locke and Mann, 2005). The disposition effect which can be caused by loss-averse behavior or by the belief that current losing positions will outperform winning positions over time (Odean, 1998) has been usually attributed to loss aversion. The role of probability weighting found here suggests that anomalies like the disposition effect may need to be re-examined in terms of both probability weighting and loss aversion. Such investigations would be in the spirit of Blavatskyy and Pogrebna (2005) and Langer and Weber (2005), and also consistent with Barberis and Thaler's (2003) call for a more integrated assessment of behavioral phenomena.

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